Geographic Routing in 3D

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ABSTRACT

Existing geographic routing algorithms assume a two-dimensional topology. Dedicated wireless sensor network scenarios demand for algorithms that operate in three-dimensional environments. This paper discusses issues which arise when making the step from 2D to 3D. Simulation studies show that 3D routing is less efficient than its 2D counterpart when comparing topologies with the same average node degree.

1. INTRODUCTION

Geographic routing algorithms forward messages using location information rather than node addresses. Each node is aware of its position via GPS or some localization algorithm. The location of the destination is included in the message, and the nodes route the message to the destination using their location information. This is especially relevant for wireless sensor networks, where the network topology is unknown a priori and subject to change, radio communication is unreliable, and position is more important than node IDs.

Existing geographic routing algorithms assume Euclidean two-dimensional topologies. Location information is held as x and y coordinates. This is sufficient when the network is deployed in a plane, e. g., for environmental monitoring in a large area. In some cases, however, the network can become three-dimensional. Application scenarios include networks within buildings, underwater networks, or even networks in space. Especially underwater sensor networks gained research interest recently [2, 8]. To enable 3D routing, the existing algorithms have to be extended. However, it is not sufficient to just add the z coordinate. This paper describes the difficulties that arise when making the step from 2D to 3D with a special focus on the Blind Geographic Routing (BGR) algorithm, which proved to come with very small communication overhead and be robust against failures, location errors, and radio irregularity.

Current research on three-dimensional networks is focused on connectivity and coverage [1, 3, 9]. Little work has been published that addresses 3D routing. In [8], two routing algorithms for underwater networks are proposed based on link metrics. The first algorithm, which is delay-insensitive, only selects nodes that are closer to the destination, and hence is subject to fail for sparse networks. The second algorithm is delay-sensitive, but uses a centralized approach. A heuristic variant of face routing, which does not guarantee delivery, but has been shown to perform well in simulations, is proposed in [4]. Some challenges in designing 3D algorithms are presented in [7].

This paper is organized as follows: Section 2 provides a short overview of the BGR algorithm. The problems when making the step toward 3D are discussed in Section 3. Simulation results for 2D and 3D topologies are presented in Section 4. Section 5 concludes the paper.

2. OVERVIEW OF BGR

This section provides a short overview of the Blind Geographic Routing (BGR) algorithm. For a detailed description and comparison to similar routing algorithms, see [10].

BGR is a two-dimensional beacon-less geographic routing algorithm using a broadcast-based contention scheme. The nodes do not carry any neighborhood or topology information. Packets are forwarded via broadcast. Nodes which receive this broadcast determine if they are located within a special area called *forwarding area*. A description of the forwarding area is included in the packet. The forwarding area is oriented toward the destination location, and its dimension ensures that all nodes within it can mutually communicate with each other (provided the unit disk graph model; however, BGR also performs well with more realistic, irregular radio propagation). Examples for forwarding areas are shown in Figure 1.

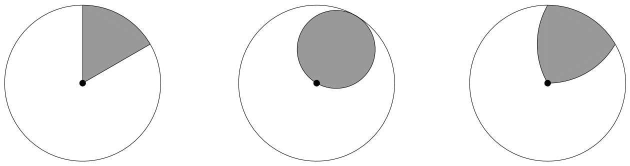

Figure 1: Forwarding areas: 60° sector, circle, and Reuleaux triangle

Nodes which receive a broadcast and are located within the forwarding area start a contention timer depending on their distance to the destination. The timer of the node which is closest to the destination expires first; this node declares itself as next hop and forwards the packet again. The other nodes which have still a timer running also receive this packet and cancel their timers.

To detect empty forwarding areas, the forwarder starts a recovery timer which is scheduled to expire after the last possible contention timer of any node within the forwarding area has expired. When the recovery timer expires, the forwarder turns the forwarding area by 60° in an arbitrary direction and broadcasts the packet again. If this forwarding area is also empty, it is turned in the other direction. When this third attempt also fails, the message is considered undeliverable and dropped. Simulation results show that this is unlikely to happen when the network density is not too low.

3. THE STEP FROM 2D TO 3D

When making the step from two to three dimensions, it is not sufficient to simply add z coordinates to location descriptions. In the following, the major issues of the transition to 3D are discussed.

Most beacon-based geographic routing algorithms perform some variant of face routing [5, 6]. The faces to be traversed are determined by the line from source to destination. However, in 3D graphs, this line does not determine the faces [4]. Thus, 2D face routing algorithms are not directly applicable to 3D. It is not known if a distributed 3D face routing algorithm exists.

Beacon-less algorithms, on the other hand, can easily be extended to operate in 3D space. The forwarding areas have to be converted into forwarding volumes by constructing the solid of revolution around the forwarder-destination axis. Hence, the 2D sector becomes a spherical sector, the circle becomes a sphere, and the Reuleaux triangle becomes the solid of revolution of a Reuleaux triangle. Note that this is different from the Reuleaux tetrahedron, whose diameter is slightly larger than the radius of the intersecting spheres from which it is constructed.

A problem with three-dimensional topologies is that more nodes are needed for network coverage than in two-dimensional topologies. For a quantitative comparison, suppose that the same average number of neighbors is to be achieved in a 2D topology of area a^2 and a 3D topology of volume a^3. The transmission range r is fixed, i. e., the unit disk graph (or, in 3D, unit sphere graph) model is assumed. If the total number of nodes in the 2D topology is n, then the average number of neighbors is $\frac{n\pi r^2}{a^2}$ (ignoring border effects). For this value to be the average number of neighbors in the 3D topology, the total number of nodes has to be

$$\frac{n\pi r^2}{a^2} \cdot \frac{a^3}{\frac{4}{3}\pi r^3} = \frac{3an}{4r}.$$

This means that in a 3D topology, the total number of nodes has to be by a factor of $\frac{3a}{4r}$ larger than in a 2D topology.

Even when the number of nodes is increased according to these calculations, a remaining problem is the additional dimension of the destination location, which leads to more possible routing directions, which result in lower delivery rates. This can easily be seen when considering the fraction of the transmission area/volume that is covered by the forwarding areas/volumes. Table 1 indicates that in 3D, the forwarding volumes cover only half as much of the transmission volume as the corresponding forwarding areas in 2D. As a consequence, the 3D version of BGR performs recovery up to four times per hop in contrast to two times in the 2D version. The first forwarding volume is obtained by turning the forwarder-destination axis by 60° in an arbitrary direction first, then in the opposite direction (mirrored about this axis), then turned by 90° about this axis, and in the last try mirrored again. A problem is that there are gaps between the turned forwarding volumes, which is not the case for the 2D forwarding areas (with the exception of the circle, which leaves two small gaps). Also, the overlapping regions are larger than in the 2D case.

Table 1: Sizes of forwarding areas/volumes as fraction of transmission area/volume

	Sector	Circle	Reuleaux triangle
	(Sph. sector)	(Sphere)	
2D	$\frac{1}{6} \approx 0.167$	$\frac{1}{4} = 0.25$	$\frac{1}{2} - \frac{\sqrt{3}}{2\pi} \approx 0.224$
3D	$\frac{1}{2} - \frac{\sqrt{3}}{4} \approx 0.067$	$\frac{1}{8} = 0.125$	$\frac{1}{2} - \frac{\pi}{8} \approx 0.107$

4. SIMULATION RESULTS

Simulation experiments have been conducted using the network simulator ns-2 to compare the performance of BGR in 2D and 3D topologies. The experiments were run with 150 nodes and a sink in the center of the topology; each node generates a data packet and sends it to the sink. The transmission range is 40 m. The average node degree was varied between 10 and 40 by adjusting the size of the (square or cubic) topology. Each value represents the average of 20 simulation runs. The Reuleaux triangle was used as forwarding area/volume.

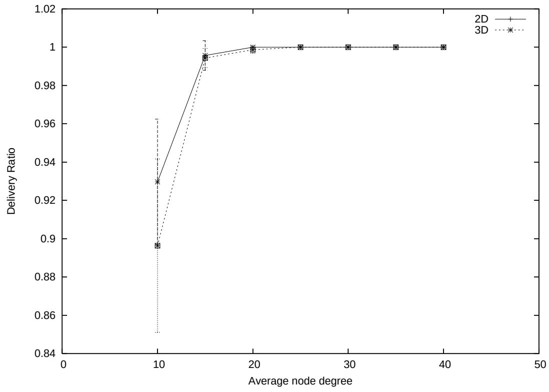

Figure 2: Delivery ratio in 2D and 3D topologies

Figure 2 shows the average delivery ratio. The error bars depict confidence intervals of 95 %. As expected, 2D routing performs better than 3D routing; the difference is most evident at low node degrees. At medium and high node degrees, however, both 2D and 3D routing perform very well and achieve constant delivery ratios of 100 %.

The total number of sent packets is depicted in Figure 3. At medium and low node degrees, 3D routing needs significantly more packets than 2D routing, which is an indicator that much more recovery is performed in 3D. The difference vanishes at high node degrees.

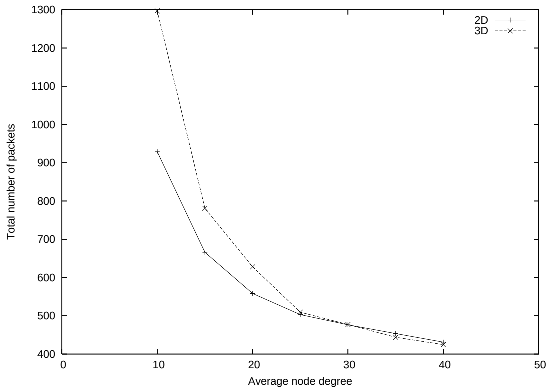

Figure 3: Number of packets in 2D and 3D topologies

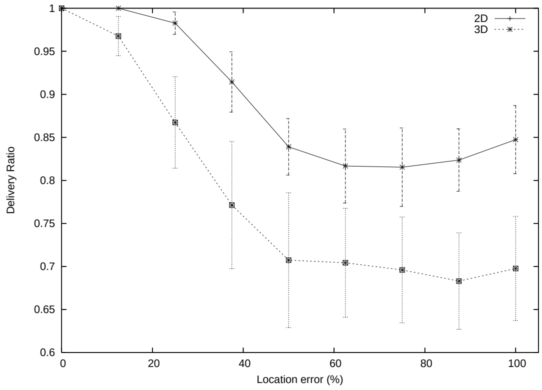

Figure 4: Delivery ratio in the presence of location errors (standard deviation as fraction of transmission range), topologies with average degree 30

As a result, the simulation studies show that the performance blockers identified in Section 3 are only relevant for low node degrees, where also 2D topologies have delivery ratios under 100 %. When the node degree is above 25, both 2D and 3D routing perform similarly well.

Location errors have a negative impact on routing performance. The impact of location errors on BGR has been studied in [11]. The error has been modeled using a two-dimensional Gaussian distribution with mean zero and standard deviation between zero and the transmission range. For 3D routing, the location error can be modeled using a three-dimensional Gaussian distribution. Figure 4 shows the corresponding simulation results, which indicate that 3D topologies are significantly more vulnerable to location errors than 2D topologies. Even small errors lead to a noticeable decrease of the delivery ratio. The major reason is that recovery is not performed when the forwarding volume is empty and the destination is assumed to be within transmission range, but due to location errors, it is not. In 2D topologies, this situation occurs less often because of the larger coverage of the the forwarding areas (cf. Table 1). Another issue is that in 3D topologies the average distance between the real and the estimated location is greater than in 2D topologies with the same standard deviation σ. In 2D, the distance follows a Rayleigh distribution, in 3D a Maxwell-Boltzmann distribution. The expected value is $\sqrt{\frac{\pi}{2}}\sigma \approx 1.253\sigma$ in 2D and $\sqrt{\frac{8}{\pi}}\sigma \approx 1.596\sigma$ in 3D.

5. CONCLUSION

Three-dimensional geographic routing is an important technique for future wireless sensor network scenarios. However, existing algorithms support merely the 2D case. The step toward 3D routing imposes some fundamental problems. Beacon-based algorithms which operate on a variant of face routing cannot simply be adopted for 3D topologies because the techniques are not directly applicable to 3D graphs. Beacon-less algorithms like BGR, on the other hand, can easily be enhanced to a 3D version by defining forwarding volumes instead of forwarding areas.

An inherent problem of 3D topologies is that more nodes are required to achieve a similar node coverage for topologies with the same average number of neighbors. BGR's forwarding volumes suffer from an analogical problem, since the covered fraction of the transmission volumes is only half as large as in the 2D case. Hence, recovery mode is triggered more often and the number of sent packets increases. The delivery ratio is also slightly lower. Additionally, simulation revealed that in case of location errors, 3D routing has significantly more problems than its 2D counterpart.

6. REFERENCES

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